This exam consists of 4 problems. Please write your answers clearly and legibly. Partial credit can be awarded, but only if you clearly show your thinking. Single-word or single-number answers, if wrong, will receive no partial credit. This is a closed book exam, though you are allowed one page of equations (8x10.5”, single-sided). No calculators are required or allowed for this exam.

Name: __Solution Set____________

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1. (25 pts) Plot the permittivity, real and imaginary, as a function of frequency/energy, resulting from optical phonons in a crystal. Label any points of interest on the frequency. Assume the same frequency scale for the plots.
   a) (10pts) Permittivity from Optical Phonons

![Graphs of real and imaginary parts of permittivity as a function of frequency.](image)

b) (5 pts)
   i. Circle the region(s) of frequency space on the (i) plot where the material above does not transmit light.
   ii. Indicate with arrows, on the (i) plot, the frequency(ies) where the material does not reflect light.

c) (10pts)
   i. (5pts) (Left) Draw the dispersion relations for acoustic and optical phonons for a 1D chain of atoms. Mark the position on the dispersion curves corresponding to energies/wavevectors which can couple to E&M radiation.
   ii. (5pts) (Right) Draw the dispersion relation for a Phonon-Polariton. Mark the light-like (A), phonon-like (B) and polariton or hybrid-like (C) portions of the dispersion curve.
2. (25pts) I optically pump a semiconductor with a laser above the semiconductor band-gap. Circle the correct answer. You can add a brief explanation of your response (1 sentence max):
   a. (3pts) Spontaneous emission lifetime increases/decreases/remains the same with increasing pump power.
   Spontaneous emission lifetime does not depend on pump power, or excited carrier concentration
   b. (3pts) Spontaneous emission intensity increases/decreases/remains the same with increasing pump power.
   Spontaneous emission rate, however, depends on how many excited carriers you have!
   c. (3pts) The rate of transitions resulting from stimulated emission increases/decreases/remains the same with increasing pump power.
   Stimulated emission rate depends on both the number of carriers excited AND the number of photons emitted by recombination of these carriers. Both will increase with increasing pump intensity.
   d. (3pts) The total transition rate of electrons from the conduction to valence band of the semiconductor is greater than/equal to/less than/can’t say than the combined transition rates from stimulated and spontaneous emission.
   Non-radiative recombination will increase transition rate from that of just radiative transitions.
   e. (3pts) The semiconductor’s absorption coefficient ($\alpha$) increases/decreases/remains the same with increasing pump intensity.
   The more carriers you excite into the conduction band, the lower your absorption coefficient will be.
   f. (3pts) The wavelength at which I observe the peak photoluminescence intensity from the sample changes with changing laser pump power (T/F).
   As you excite carriers, you fill up increasingly higher energy states, this will blueshift your emission.
   g. (4pts) Increasing the laser frequency (but maintaining the same photon flux) will increase/decrease/not change the electron hole pairs generated in the first 10nm of the semiconductor. Briefly explain.
   Absorption coefficient goes as $\alpha(\omega) \propto \sqrt{\hbar \omega - E_g}$, so as frequency increases, absorption will increase.
   h. (3pts) Electron and hole populations in this system can be described by a single Fermi level $E_F$ (T/F).
   This is a non-equilibrium system, so must use quasi Fermi levels for electrons and hole.
3. (25pts) A two-level system has eigenenergies $E_1 = 3\hbar \omega_1$ and $E_2 = 5\hbar \omega_1$ sits in a cavity with three allowed modes, of frequencies $\omega_1$, $2\omega_1$, and $3\omega_1$. The total eigenfunction for the system can be described, in bra-ket notation, by the state of the electron ($|1\rangle$ or $|2\rangle$), followed by the Fock states for each mode of the cavity ($|n_1,n_2,n_3\rangle$), giving $|1$ or $2, n_1, n_2, n_3\rangle$.

a. At $t = 0$, the system is in the state described by $|1, 4, 2, 6\rangle$.
   i. (4pts) What is the eigenvalue for the E&M Hamiltonian ($H_{rad}$) acting on the system?

   \[
   \sum_k \hbar \omega_k \left( a_k^\dagger a_k + \frac{1}{2} \right) |1, 4, 2, 6\rangle = \sum_k \hbar \omega_k \left( n_k + \frac{1}{2} \right) |1, 4, 2, 6\rangle \\
   \hbar \omega_1 \left( 4 + \frac{1}{2} \right) + \hbar 2\omega_1 \left( 2 + \frac{1}{2} \right) + \hbar 3\omega_1 \left( 6 + \frac{1}{2} \right) = 29\hbar \omega_1
   \]

   ii. (3pts) The electron in the system absorbs a photon. Give the final state of the system.

   $|2, 4, 1, 6\rangle$

b. At $t = 0$ the initial state of the system is $|2, 3, 0, 4\rangle$.
   i. (5pts) What is the eigenvalue for the E&M and electron Hamiltonians ($H_e + H_{rad}$) acting on the system?

   \[
   (H_{rad} + H_e)|\Psi\rangle = \left( \sum_k \hbar \omega_k \left( a_k^\dagger a_k + \frac{1}{2} \right) + H_e \right) |2, 3, 0, 4\rangle \\
   \hbar \omega_1 \left( 3 + \frac{1}{2} \right) + \hbar 2\omega_1 \left( 0 + \frac{1}{2} \right) + \hbar 3\omega_1 \left( 4 + \frac{1}{2} \right) + 5\hbar \omega_1 = 23\hbar \omega_1
   \]

   ii. (4pts) You observe the emission of a photon into the cavity. Is this emission more likely to be a spontaneous or stimulated emission event? Explain.

   *There are no photons in the cavity with energy $2\hbar \omega_1$, therefore there is nothing to stimulate a transition from $|2\rangle$ to $|1\rangle$. Spontaneous.*

c. At $t = 0$, the system is in the state described by $|2, 3, 6, 4\rangle$.
   i. (4pts) You observe the emission of a photon into the cavity. Is this emission more likely to be a spontaneous or stimulated emission event? Explain.

   *Downward transition rate is proportional to $n_k + 1$, where the $n_k$ refers to stimulated transitions and the 1 to spontaneous. Since $n_k = 6 > 1$, transitions will be dominated by stimulated processes.*

   ii. (5pts) You measure the occupation of the system’s excited state as a function of time and notice that the state periodically becomes occupied and then empties again. Name/explain the phenomenon you are observing.

   *Rabi Oscillations: coherent oscillations of energy between cavity and 2 level quantum mechanical system.*
4. (25pts) Suppose I have two undoped direct band–gap semiconductors, A and B. Semiconductor A has \( E_{Ag} = 1.5 eV, m_A^* = 0.1, m_A^* = 0.5 \), and semiconductor B has \( E_{Ag} = 1.8 eV, m_B^* = 0.5, m_B^* = 0.5 \).

a. (8 pts) Below, sketch the electron dispersion relations (E vs k) for each semiconductor.

b. (8 pts) Plot the absorption of each semiconductor (A and B) below. Include excitonic effects, and note the absorption resulting from i) bound and ii) free excitons.

c. (9 pts) If I make a sandwich of my materials B/A/B, where A has a thickness of ~3nm, plot the (I) absorption and (II) PL emission from this structure as a function of energy, ignoring excitonic effects.